

Q1.  $-12(y+2) = (x-4)^2$

i) sketch

\* directrix is horizontal

ii) vertex

$4d = -12$

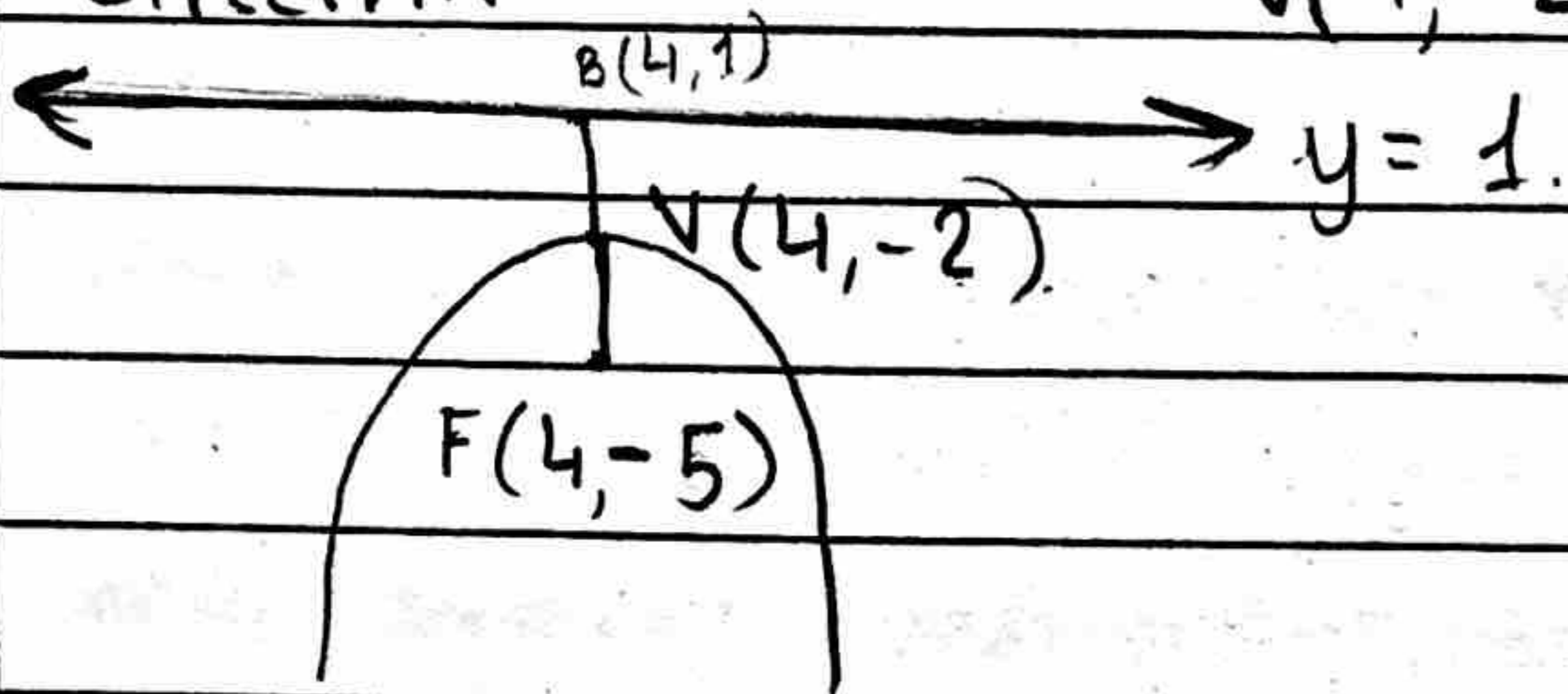
iii) focus.

$d = -3 < 0 \Rightarrow$  open down.

iv) directrix.

$V(4, -2)$

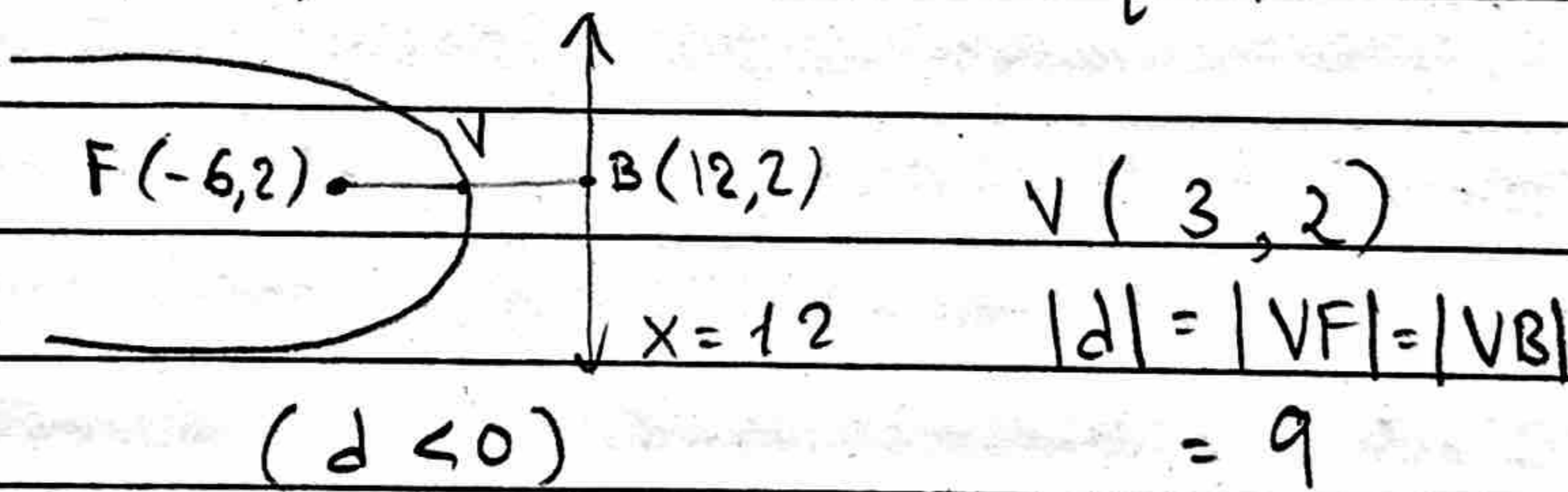
i)



- ii)  $V(4, -2)$     iii)  $F(4, -5)$     iv)  $y = 1$ .

Q2.  $F(-6, 2)$ ; directrix  $x = 12$     eqn = ?

$4d(x-x_0) = (y-y_0)^2$



$(d < 0)$

$V(3, 2)$

$|d| = |VF| = |VB|$

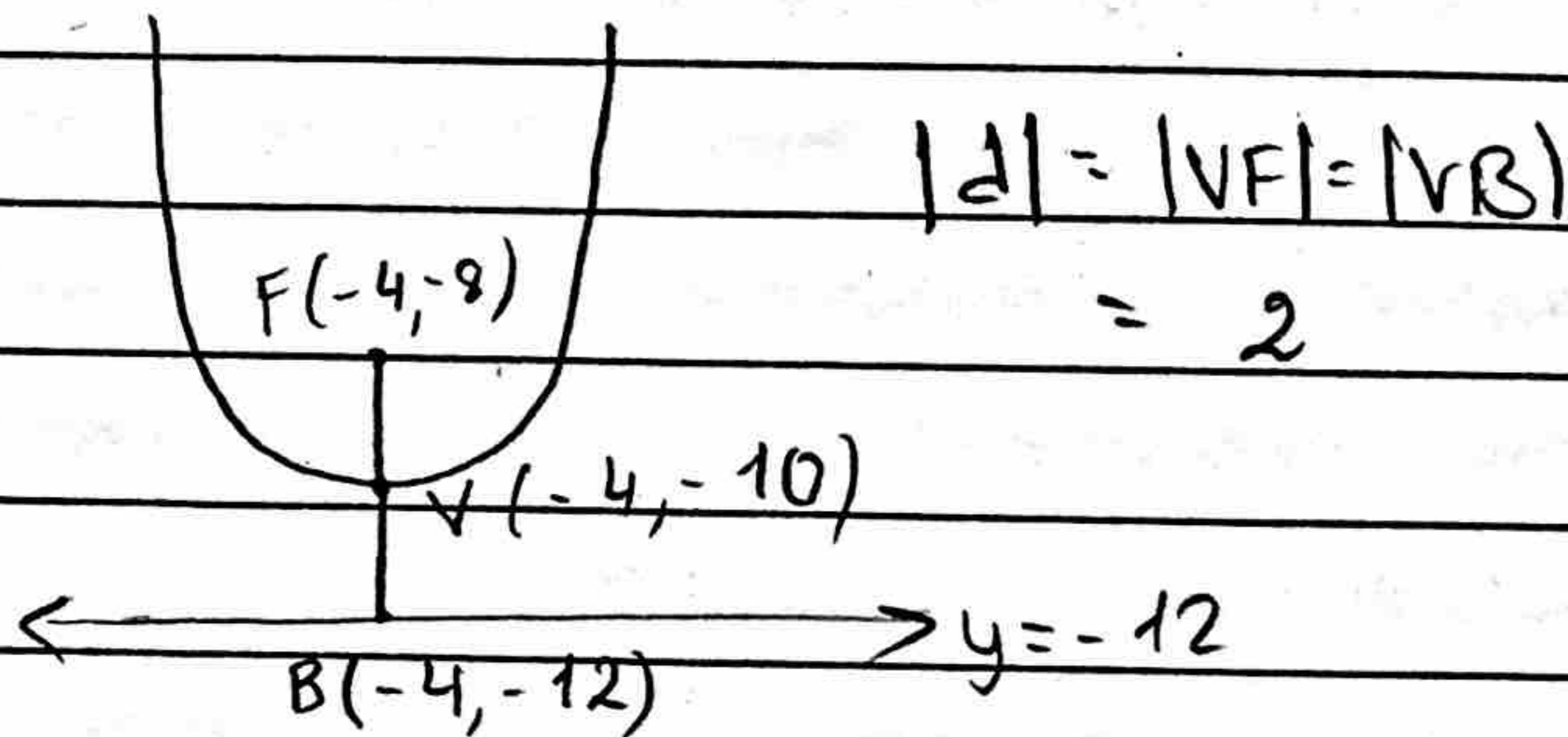
$= 9$

$\Rightarrow -36(x-3) = (y-2)^2$

Q3.  $F(-4, -8)$     eqn = ?

directrix  $y = -12$

$4d(y-y_0) = (x-x_0)^2$



$|d| = |VF| = |VB|$

$= 2$

$\Rightarrow 8(y+10) = (y+4)^2$

$(d > 0)$

Questions

1-  $2y = 3x^2 + 12x + 4$

- a) write in SF
- b) sketch
- c) find focus, vertex and directrix line

2-  $x = -5y^2 - 20y + 12$

- a) write in SF
- b) sketch
- c) find focus, vertex and directrix line

Solution

1- a)  $2y = 3x^2 + 12x + 4$   
 $2y = [3(x^2 + 4x)] + 4$   
 $2y = [3(x^2 + 2x + 2)^2 - 4] + 4$   
 $2y = 3(x + 2)^2 - 12 + 4$   
 $2y = 3(x + 2)^2 - 8$

$\frac{1}{2} (2y + 8) = 3(x + 2)^2$

$y + 4 = \frac{1}{2} [3(x + 2)^2]$

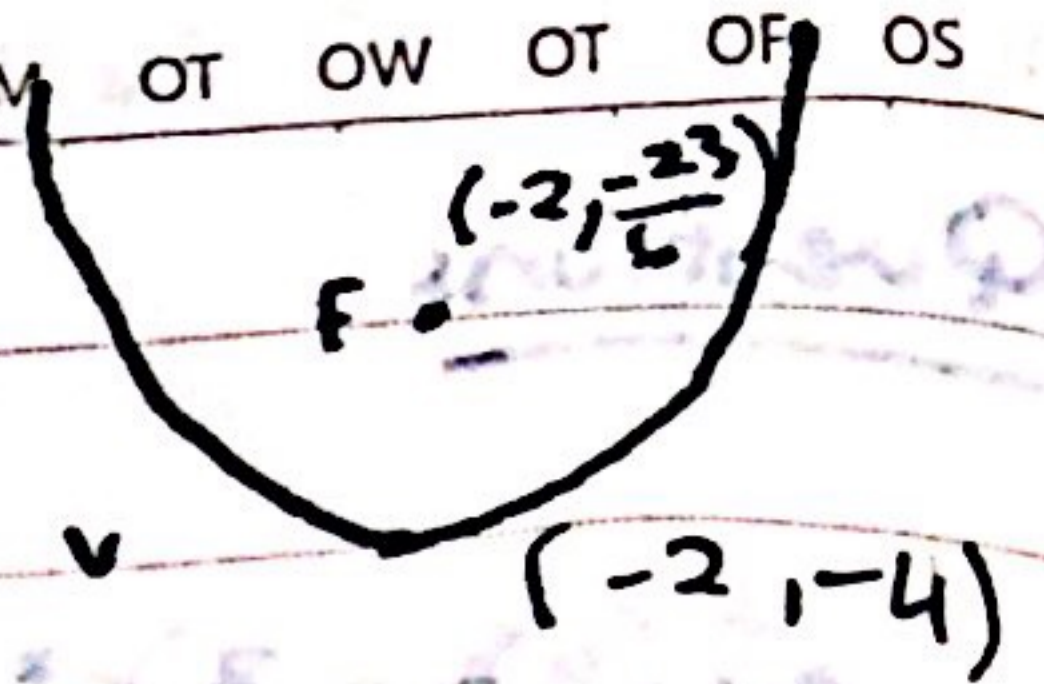
$\frac{2}{3} (y + 4) = \frac{3}{2} (x + 2)^2 \times \frac{2}{3}$

$\frac{2}{3} (y + 4) = (x + 2)^2$

Her solution is correct, but here is a diff. way

$2y = 3(x+2)^2 - 8$   
 $2y + 8 = 3(x + 2)^2$   
 $2(y + 4) = 3(x+2)^2$   
 $\frac{2}{3}(y + 4) = (x+2)^2$   
 $4d = \frac{2}{3}$ , so  $d = \frac{2}{12} = \frac{1}{6}$

Now continue ....



b- vertex =  $(-2, -4)$

$y = \frac{-25}{6}$

$4D = \frac{2}{3} \quad D = \frac{1}{6}$

Focus =  $(-2, \frac{-23}{6})$

Directrix line goes through  ~~$(-2, \frac{25}{6})$~~   $y = \frac{-25}{6}$

Q:  $x = -5y^2 - 20y + 12$

a)  $x = [-5(y^2 + 4y)] + 12$

$x = [-5(y+2)^2 - 4] + 12$

$x = -5(y+2)^2 + 20 + 12$

$x = -5(y+2)^2 + 32$

goes through x line  
up to the right

$\frac{1}{5}(x-32) = \frac{-5(y+2)^2}{-5}$

$4D = \frac{1}{5}$

$D = 0.05 = \frac{1}{20}$

$\frac{1}{5}(x-32) = (y+2)^2$

b) vertex =  $(32, -2)$

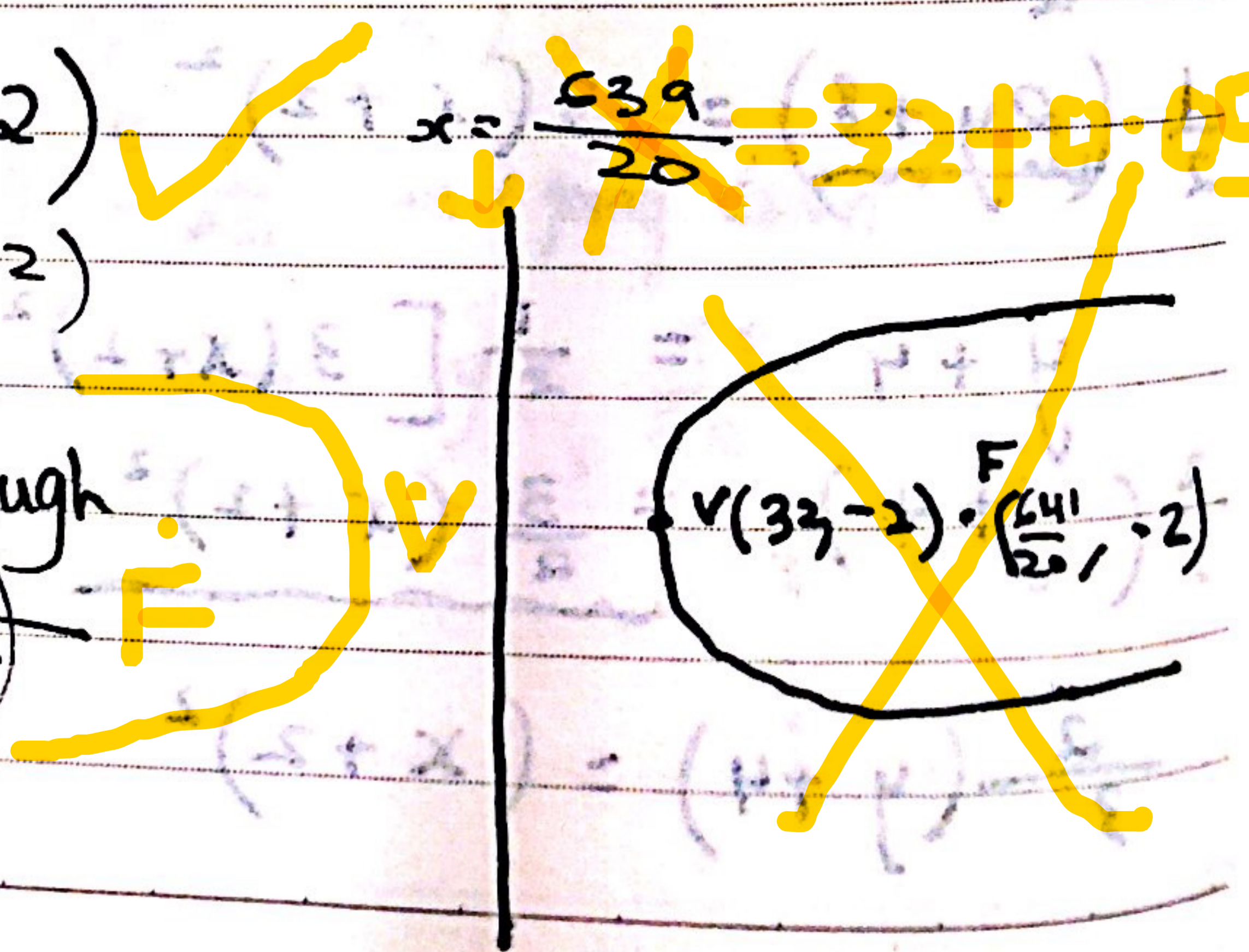
focus =  $(\frac{641}{20}, -2)$

~~$x = \frac{639}{20} = 32 + 0.05$~~

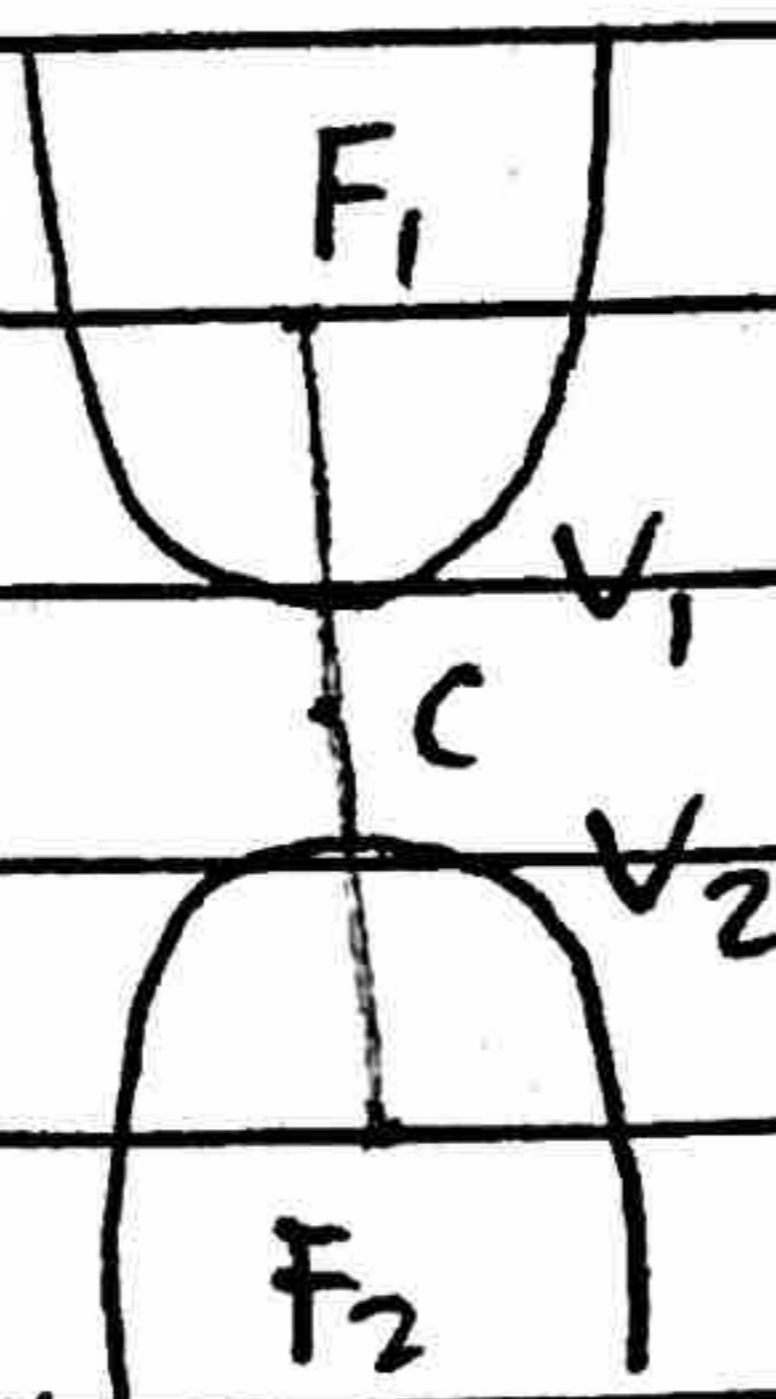
$32 - 0.05$

directrix line goes through

~~$x = (\frac{639}{20})$   $x = (\frac{639}{20} - 2)$~~



Q1.  $\frac{(y-1)^2}{16} - \frac{x^2}{9} = 1.$



\* sketch

2 - Foci ?

3 - vertices ?

4 - k ?

$(\frac{k}{2})^2 = 16 \Rightarrow k = 4$   
 $\Rightarrow \boxed{k = 8}$

$b^2 = 9 \Rightarrow b = 3.$

$C(0, 1)$

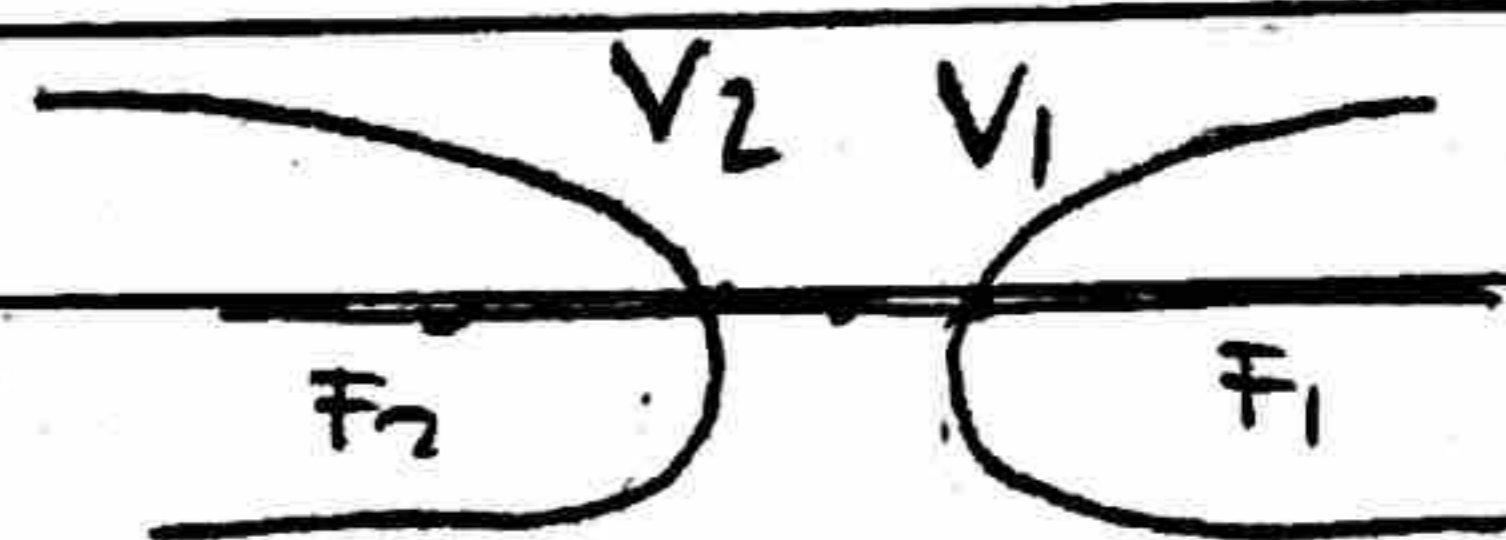
$V_1(0, 1+4) \quad V_2(0, 1-4)$

$V_1(0, 5) \quad V_2(0, -3)$

$|CF_1| = \sqrt{16+9} = 5 \Rightarrow F_1(0, 1+5) \quad F_2(0, 1-5)$

$F_1(0, 6) \quad F_2(0, -4)$

Q2.  $\frac{(x+4)^2}{3} - (y-5)^2 = 1$



$(\frac{k}{2})^2 = 1 \Rightarrow k = 1 \Rightarrow \boxed{k = 2}$

$C(-4, 5)$

$b^2 = 3 \Rightarrow b = \sqrt{3}$

$C(-4, 5)$

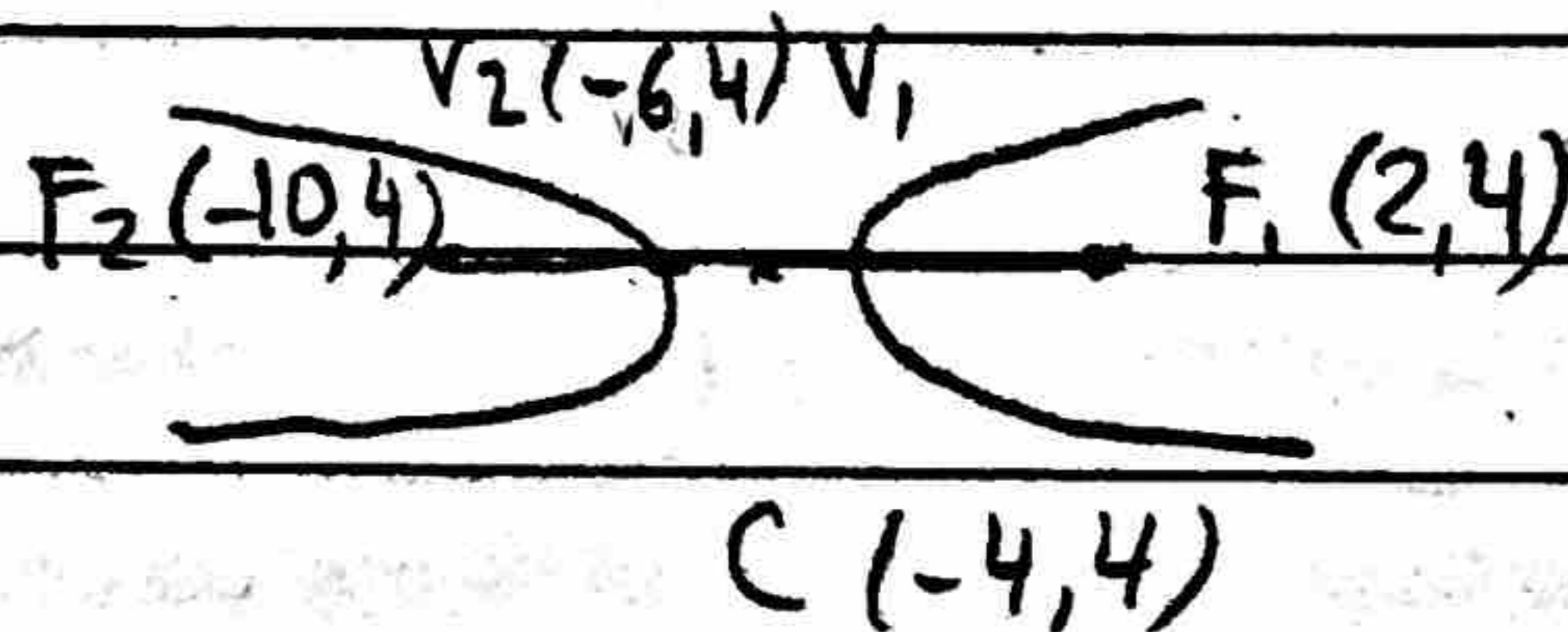
$V_1(-4+1, 5) \quad V_2(-4-1, 5)$

$V_1(-3, 5) \quad V_2(-5, 5)$

$|CF_1| = \sqrt{1+3} = 2 \Rightarrow F_1(-4+2, 5) \quad F_2(-4-2, 5)$

$F_1(-2, 5) \quad F_2(-6, 5)$

Q3.  $F_1(2, 4) \quad F_2(-10, 4) \quad V_2(-6, 4)$



a)  $k = ?$  b)  $V_1 = ?$  c) eqn = ?

$C(-4, 4)$

$|CV_2| = \frac{k}{2} = 2 \Rightarrow \boxed{k = 4}$

$|CV_1| = |CV_2| = k$

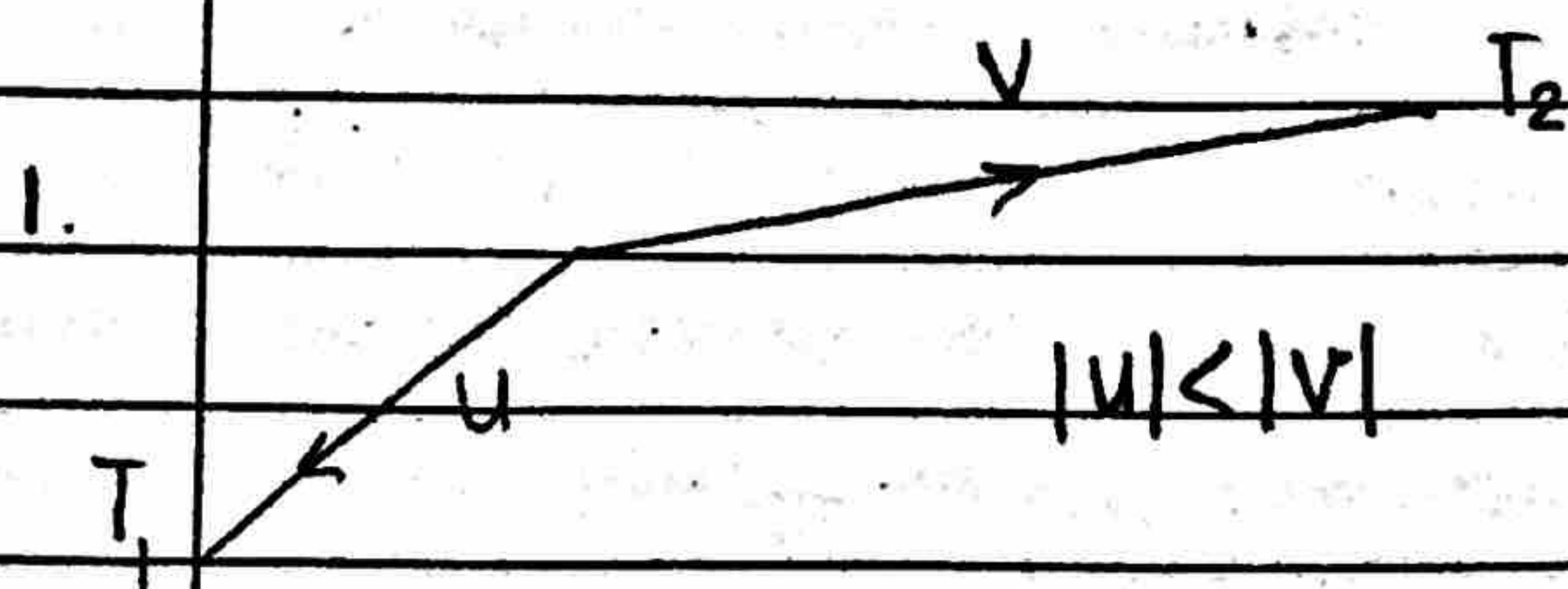
$\Rightarrow \boxed{V_1(-2, 4)}$   $b^2 = |CF_2|^2 - (\frac{k}{2})^2 = 6^2 - 2^2 = 32.$

$|CF_2| = 6$

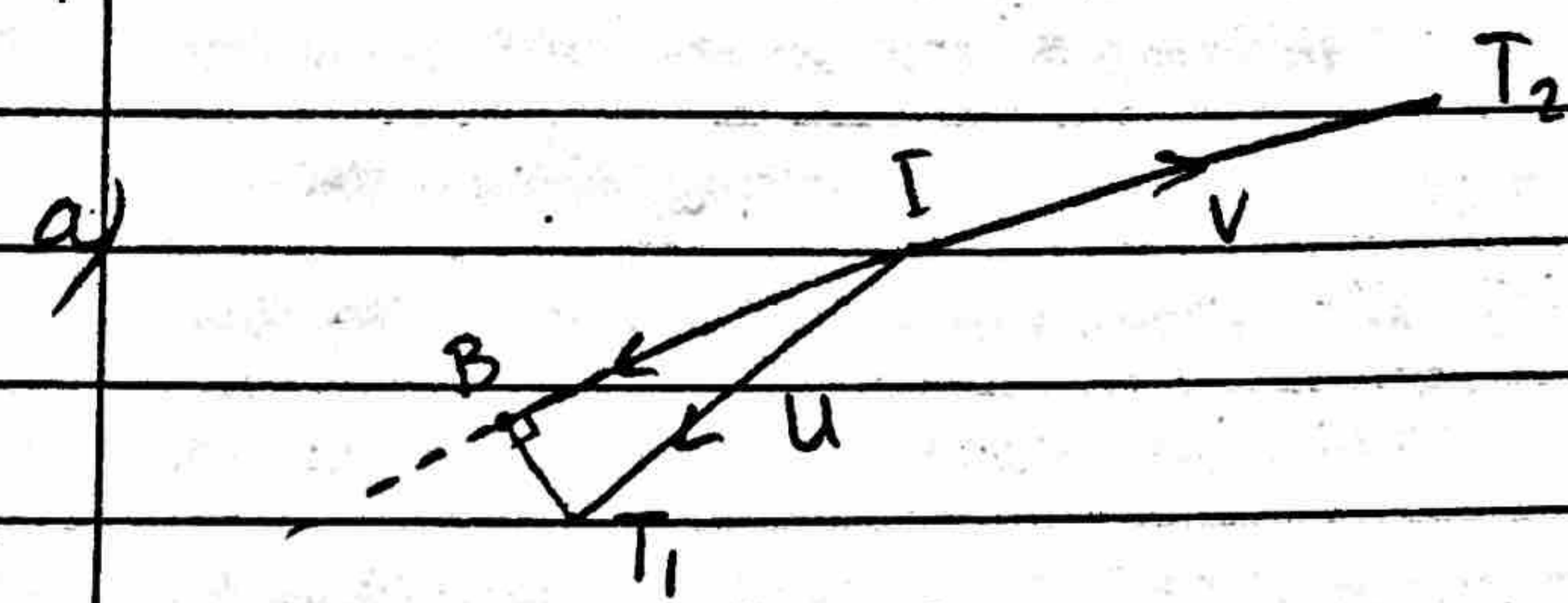
$\frac{(x+4)^2}{4} - \frac{(y-4)^2}{32} = 1$

HW 5.

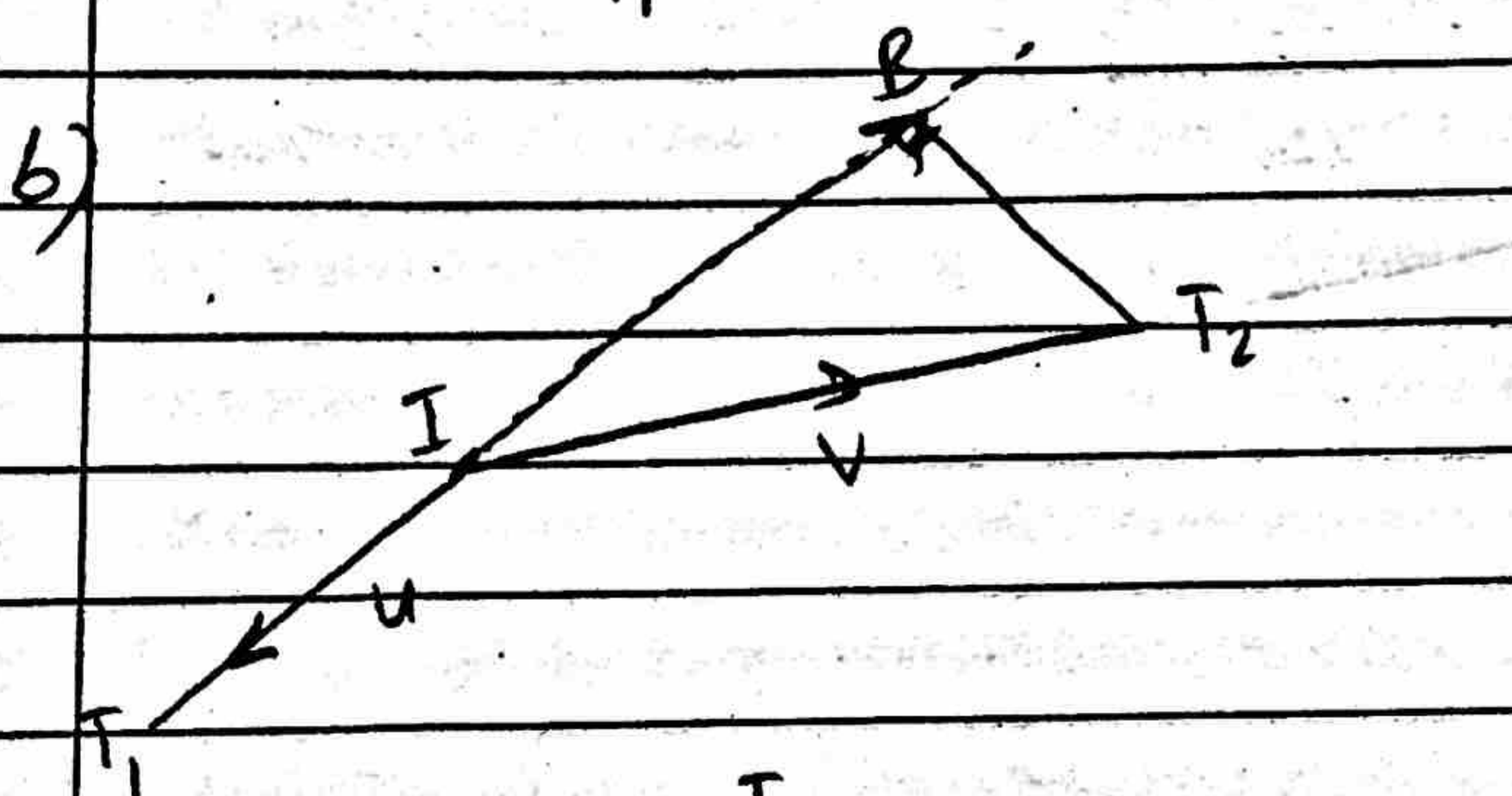
11/02/2018



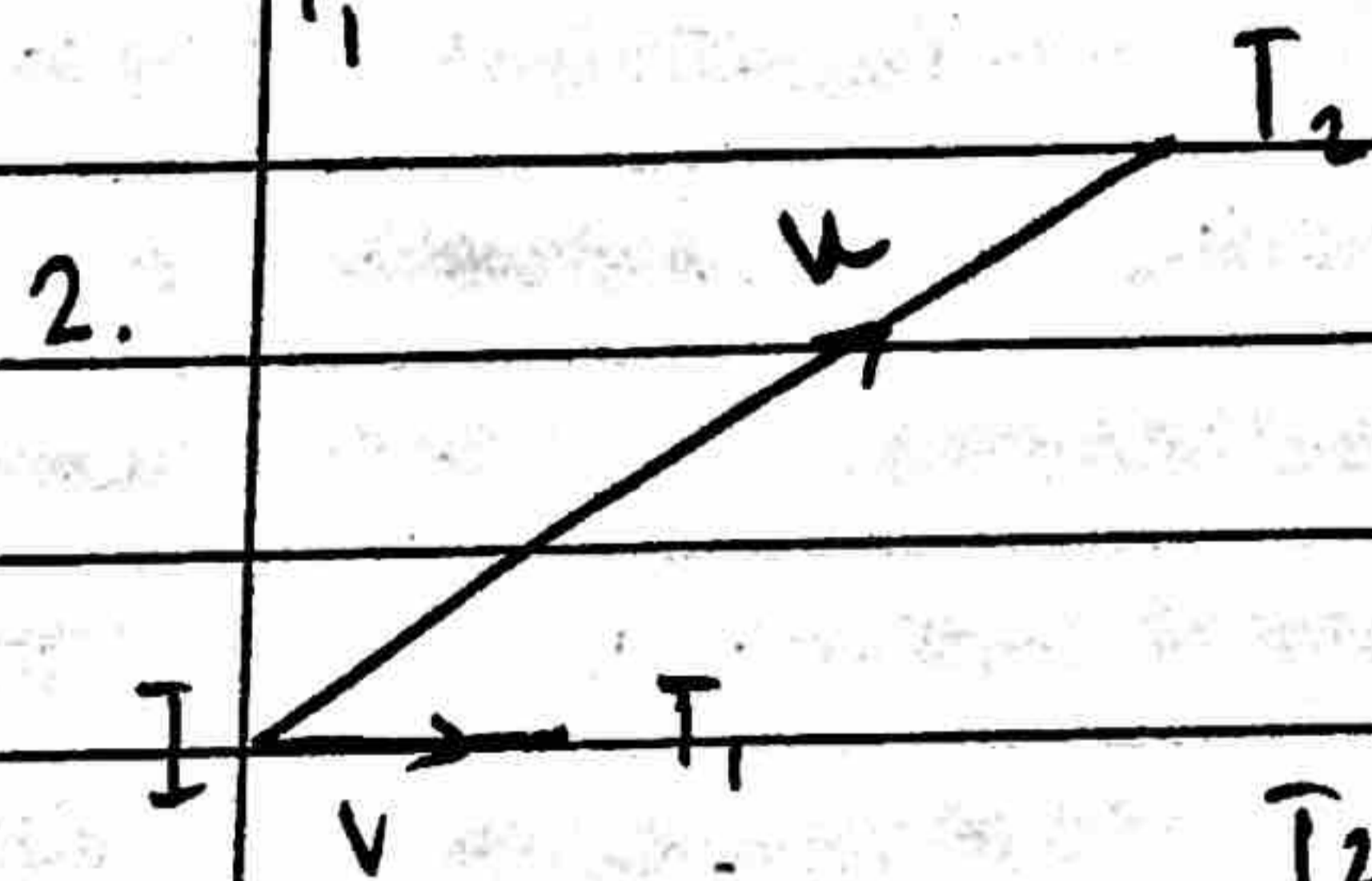
- a) Draw  $\text{proj}_v^u$
- b) Draw  $\text{proj}_u^v$



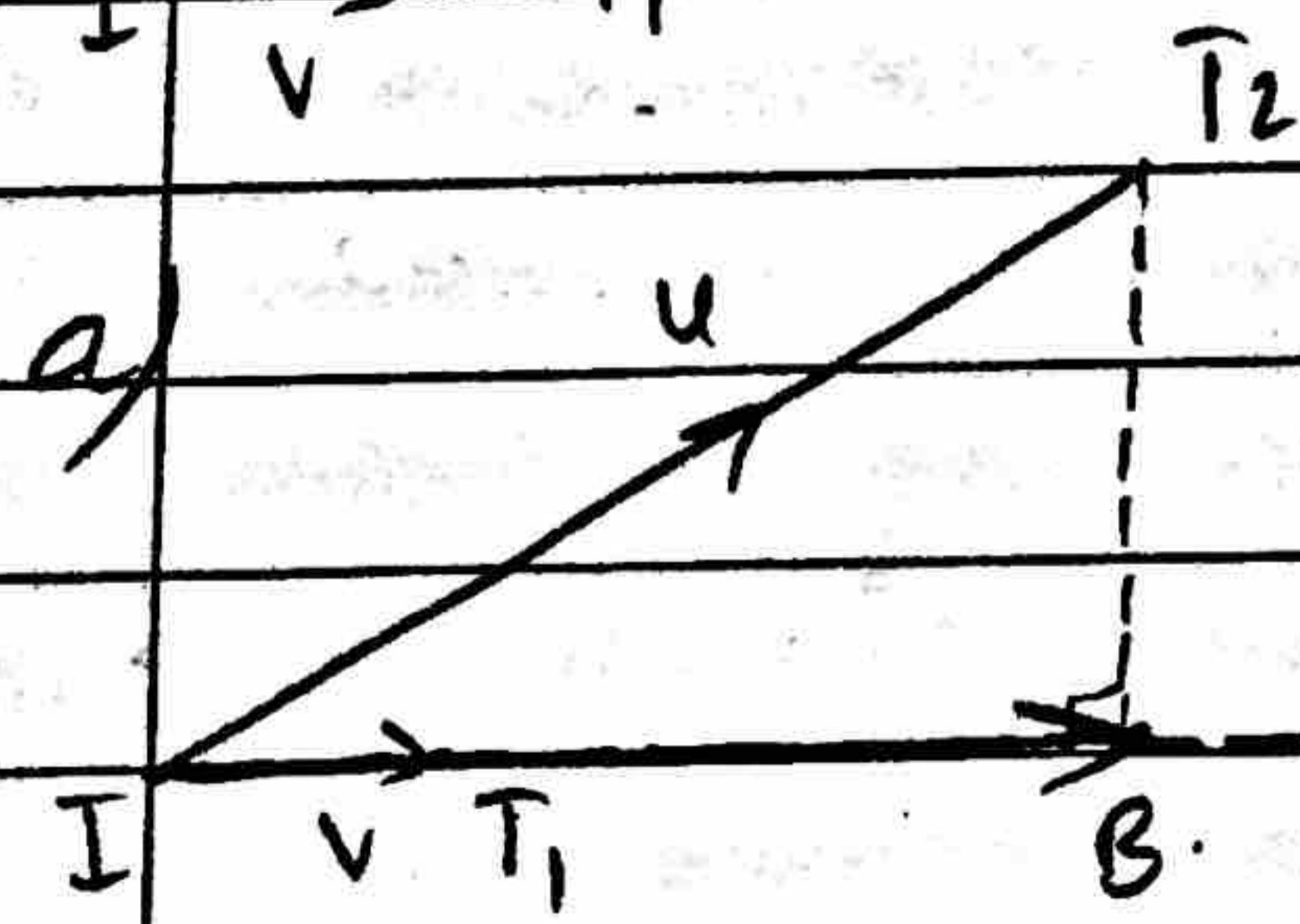
$\text{proj}_v^u = \overrightarrow{IB}$



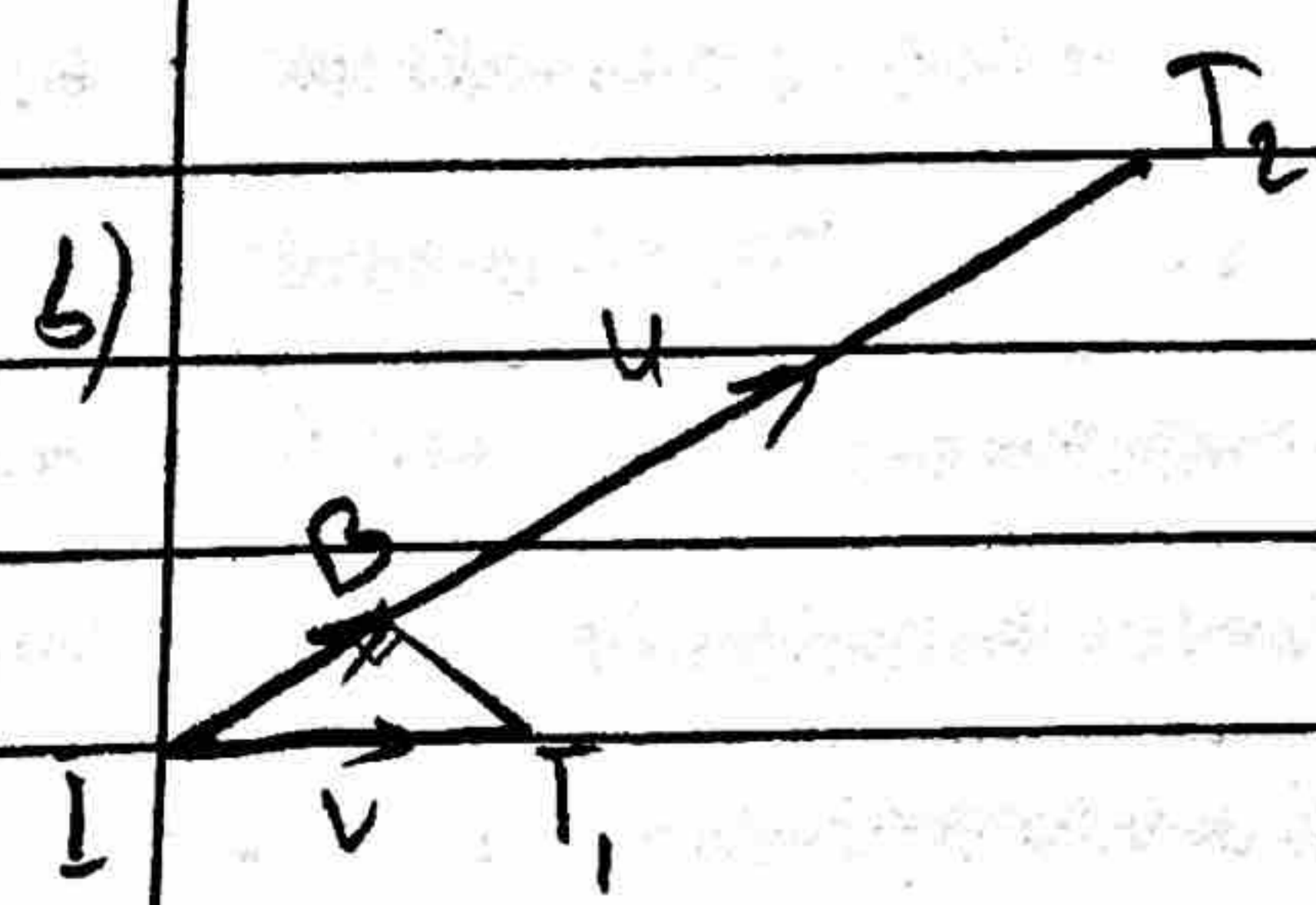
$\text{proj}_u^v = \overrightarrow{IB}$



- a) Draw  $\text{proj}_v^u$
- b) Draw  $\text{proj}_u^v$



$\text{proj}_v^u = \overrightarrow{IB}$



$\text{proj}_u^v = \overrightarrow{IB}$

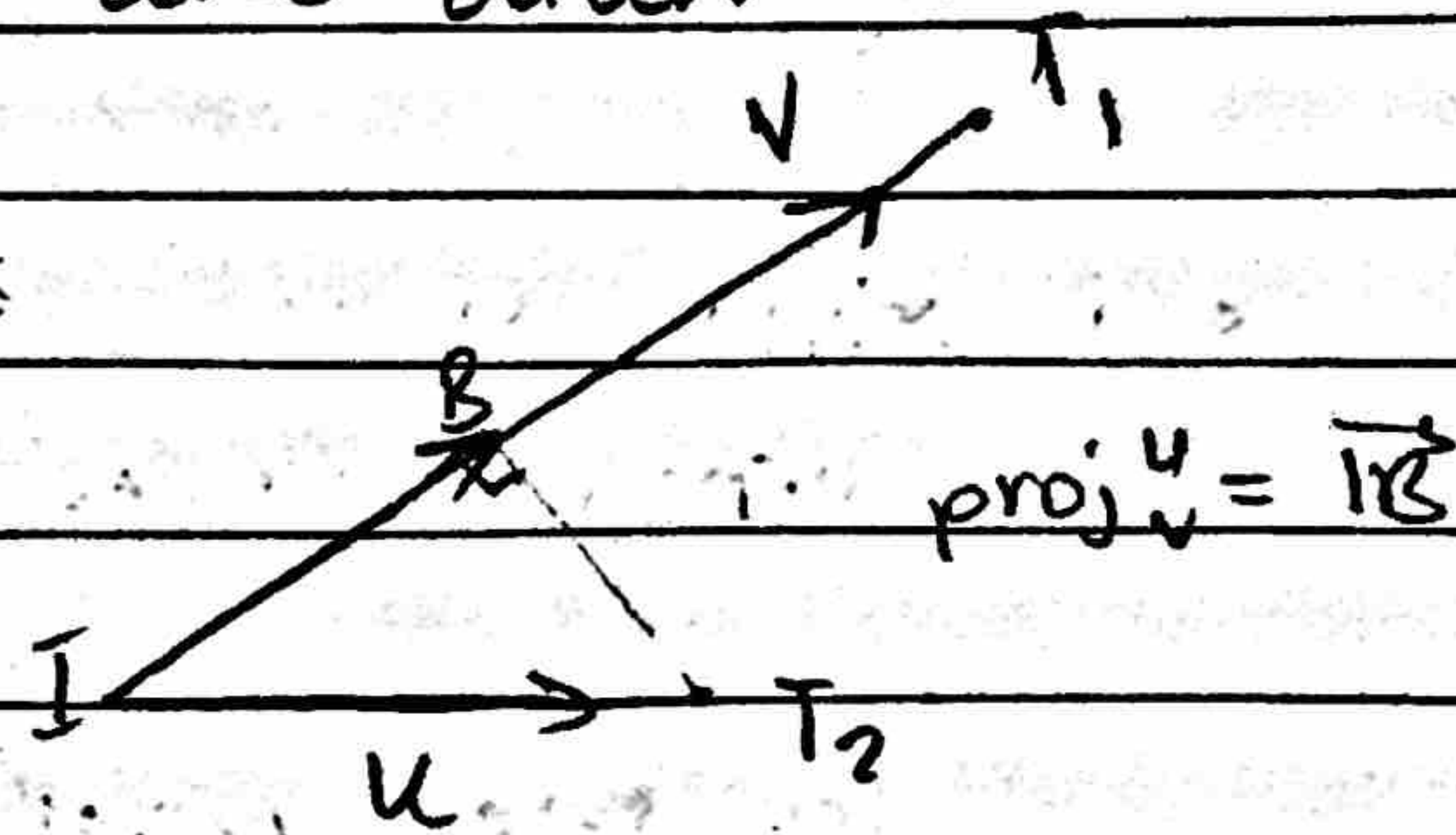
3.

$$u = \langle 1, 4, 6 \rangle$$

Find  $\text{proj}_v^u$  and draw

$$v = \langle -2, 10, 1 \rangle$$

$$\text{proj}_v^u = \frac{u \cdot v}{|v|^2} v$$



$$u \cdot v = 1(-2) + 4(10) + 6(1) = 44$$

$$|v| = \sqrt{2^2 + 10^2 + 1^2} = \sqrt{105}$$

$$\text{proj}_v^u = \frac{44}{105} \cdot v = \frac{44}{105} \langle -2, 10, 1 \rangle = \left\langle -\frac{88}{105}, \frac{88}{21}, \frac{44}{105} \right\rangle$$

## NOTES

Q Given  $\Delta = \langle -4, 7, 2 \rangle$  is the directional vector of line L and  $(2, -4, 10)$  lies on L

1. find parametric eqn of L

$$2 - 4L, -4 + 7L$$

$$x = 2 - 4L$$

$$y = -4 + 7L$$

$$z = 10 + 2L$$

$$\frac{4L}{4} = \frac{2-x}{4}$$

$$\frac{7L}{7} = \frac{y+4}{7}$$

$$\frac{2L}{2} = \frac{2-10}{2}$$

$$L = \frac{2-x}{4}$$

$$L = \frac{y+4}{7}$$

$$L = \frac{2-10}{2}$$

2. find symmetric eqn of L

$$\frac{2-x}{4} = \frac{y+4}{7} = \frac{2-10}{2}$$

3. Does  $(-2, 3, 12)$  lie on L

$$\frac{2 - (-2)}{4} = 1$$

$$\frac{3+4}{7} = 1$$

$$\frac{12-10}{2} = 1$$

yes they lie on ~~line~~ L  
on all L values  
are the same

Q. Given  $(1, 2, 4)$  and  $(-10, 3, 0)$  are on line L  
 parametric eqn =

$$(-10-1, 3-2, 0-4) = (-11, 1, -4)$$

$$\langle -11, 1, -4 \rangle \quad (1, 2, 4)$$

$$x = 1 - 11t$$

$$y = 2 - t$$

$$z = 4 + 4t$$

$$\frac{1-t}{11} = \frac{1-x}{11}$$

$$t = 2 - y$$

$$\frac{4t}{4} = \frac{z-4}{4}$$

$$t = \frac{1-x}{11}$$

$$t = \frac{z-4}{4}$$

Symmetric eqn =

$$\frac{1-x}{11} = 2-y = \frac{z-4}{4}$$



Q: Given  $w = \langle 1, 4, 7 \rangle$  and  $v = \langle -4, 2, -10 \rangle$

find  $|\text{proj}_v w|$  and  $|\text{proj}_w v|$

$$a) |\text{proj}_v w| = \frac{|w \cdot v|}{|v|}$$

$$= \frac{|-4 + 8 - 70|}{\sqrt{4^2 + 2^2 + 10^2}}$$

$$= \frac{66}{10}$$

$$= \frac{66}{10}$$

$$b) |\text{proj}_w v| = \frac{|v \cdot w|}{|w|} = \frac{66}{10}$$

$$= \frac{66}{10}$$

$$\frac{66}{\sqrt{4^2 + 1^2 + 7^2}} = \sqrt{66}$$

Leen AlNimer

• Hw Q1)  $L_1: x = 4t - 1$   
 $y = 2t + 2$   $t = R$   
 $z = 10$

$L_2: x = 3\omega - 6$   
 $y = -2\omega + 10$   $\omega = R$   
 $z = 3\omega + 1$

$$4t - 1 = 3\omega - 6$$

$$3\omega + 1 = 10$$

$$4t - 3\omega = 1 - 6$$

$$3\omega = 9$$

$$4t - 3\omega = -5$$

$$3\omega = 9$$

$$4t = 4$$

$$t = 1$$

$$4(1) - 3\omega = -5$$

$$\omega = 3$$

$$t=1 \rightarrow (3, 4, 10)$$

$$\omega=3 \rightarrow (3, 4, 10)$$

We may assume that the two airplanes took off from two different airports at the same time!! So Plane 1 will reach the point (3, 4, 10) after one hour, where Plane 2 will arrive

~~Both planes will arrive at the~~

~~same point~~

~~(3, 4, 10) at the~~

~~same time~~  
at the same point after 3 hours.

Hence no COLLISION

Q2.  $L_1: x = 4t - 1$

$$y = 2t + 2 \quad t \in \mathbb{R}$$

$$z = 4t + 3$$

$$L_2: x = 3\omega - 6$$

$$y = -2\omega + 10 \quad \omega \in \mathbb{R}$$

$$z = 3\omega + 1$$

$$4t - 1 = 3\omega - 6$$

$$4t - 3\omega = 1 - 6$$

$$4t - 3\omega = -5$$

$$-2(2t + 2\omega = 8)$$

$$2t + 2 = -2\omega + 10$$

$$2t + 2\omega = -2 + 10$$

$$4t - 3\omega = -5$$

$$-4t - 4\omega = -16$$

$$-7\omega = -21$$

$$\omega = 3$$

$$4t - 3(3) = -5$$

$$t=1$$

$$t=1 \rightarrow (3, 4, 7)$$

don't interseed

$$w=3 \rightarrow (3, 4, 10)$$

because  $7 \neq 10$ .

## HW 8: MTH 111, Spring 2018

Ayman Badawi

**QUESTION 1.** Given  $L_1 : x = 2t + 1, y = -4t + 6, z = 7t + 3$  and  $L_2 : x = 6w - 9, y = -12w + 26, z = -21w + 30$ . Convince me that  $L_1$  is not parallel to  $L_2$ . Also, does  $L_1$  intersect  $L_2$ ?

**Solution:**  $D_1 = \langle 2, -4, 7 \rangle, D_2 = \langle 6, -12, -21 \rangle$ . Set  $D_1 = cD_2$ , so  $\langle 2, -4, 7 \rangle = c \langle 6, -12, -21 \rangle = \langle 6c, -12c, -21c \rangle$ . Hence  $2 = 6c$  implies  $c = 1/3$ ;  $-4 = -12c$  implies  $c = 1/3$  (so far so good since we are getting the same value for  $c$ );  $7 = -21c$  implies  $c = -1/3$ . Whips not good, so we cannot find one value for  $c$  to make  $D_1 = cD_2$ . Hence stop,  $L_1$  is not parallel to  $L_2$ .

Let us check if  $L_1$  intersects  $L_2$ . Make  $x$  in  $L_1 = x$  in  $L_2$  and make  $y$  in  $L_1 = y$  in  $L_2$ . We get the following two equations (after moving around)

$$2t - 6w = -10$$

$$-4t + 12w = 20$$

. We multiply equation (1) with 2, then we add both equations. We get  $0 = 0$ . Bad Luck (sad face). So we set  $x$  in  $L_1 = x$  in  $L_2$  and  $z$  in  $L_1 = z$  in  $L_2$

$$2t - 6w = -10$$

$$7t + 21w = 27$$

Multiply the first equation with  $-3.5$  and then we add both equations. We get  $42w = 62$ . Hence  $w = 31/21$ . Substitute for  $w$  in one of the equations (I choose second equation), we get  $t = -4/7$ . Now if  $L_1$  intersect  $L_2$ , then  $y$ -value of  $L_1$  when  $t = -4/7$  must equal  $y$ -value of  $L_2$  when  $w = 31/21$ .

$y = -4t + 6$  in  $L_1$ , so substitute  $t = -4/7$ , we get  $y = 8.286$  (approx),  $y = -12w + 26$  in  $L_2$ , so substitute  $w = 31/21$ , we get  $y = 8.286$  (approx).

Thus  $L_1$  indeed intersect  $L_2$ . So now to find the point of intersection. Let  $t = -4/7$  find  $x, y, z$  from  $L_1$ . (you may choose  $w = 31/21$ , find  $x, y, z$  in  $L_2$ , you must get the same point). we get  $(-0.143, 8.286, -1)$

**QUESTION 2.**  $L_1 : x = 2t + 1, y = -4t + 6, z = 7t + 3$  and  $L_2 : x = 6w - 9, y = -12w + 26, z = 21w - 30$ . Convince me that  $L_1$  is parallel to  $L_2$  (Hint: Another method to solve this question (I gave it last semester, but students seem were not convinced !): First Check if  $D_1 = cD_2$  for some number  $c$ . If no, then stop and we conclude that they are not parallel. If yes, then choose a point  $Q$  randomly on  $L_1$  (Here you may choose  $Q = (1, 6, 3)$ . If  $Q$  lies on  $L_2$ , then  $L_1$  lies on top of  $L_2$  and in this case they are not parallel. 2) If  $Q$  does not lie on  $L_2$ , then  $L_1$  is parallel to  $L_2$ .

**Solution:**  $D_1 = \langle 2, -4, 7 \rangle$  and  $D_2 = \langle 6, -12, 21 \rangle$ . Set  $D_1 = cD_2$ . One can conclude that  $c = 1/3$ . Thus  $D_1$  is parallel to  $D_2$ . So we continue. Let  $t = 0$  in  $L_1$ . We get the point  $Q = (1, 6, 3)$ . Now check if  $Q$  lies on  $L_2$ . Set  $1 = 6w - 9$ , we get  $w = 10/6 = 5/3$ . Set  $6 = -12w + 26$ , we get  $w = 20/12 = 5/3$  (so far so good, we getting the same value for  $w$ ). Set  $3 = 21w - 30$ , we get  $w = 33/21 = 11/7$ , whips... not good. So  $L_1$  is parallel to  $L_2$

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1.  $L_1: \begin{cases} x = 2t + 1 \\ y = -4t + 2 \\ z = 7t + 3 \end{cases}$       $L_2: \begin{cases} x = 6w - 17 \\ y = -12w + 38 \\ z = 21w + 2 \end{cases}$

$Q(1, 2, 3) \in L_1$       $D_2 = \langle 6, -12, 21 \rangle$   
 $V \times D = \langle 744, -342, 408 \rangle$   
 $|QL_2| = \frac{|L_1 \cdot L_2|}{|D|} = \frac{|V \times D|}{|D|} \approx 36.71 \text{ units}$       $|V \times D| \approx 914.86$   
 $|D| = 3\sqrt{69}$

2.  $L: \begin{cases} x = 6w - 17 \\ y = -12w + 38 \\ z = 21w + 2 \end{cases}$       $Q(7, 2, 2)$      Choose a point I on L. Say (-17, 38, 2). Then QI = V as below, note that if you do IQ, then you get the vector <24, -36, 0>. This will not affect the final answer

$D = \langle 6, -12, 21 \rangle$       $V = \langle -24, 36, 0 \rangle$   
 $V \times D = \langle 756, 504, 72 \rangle$       $|V \times D| = 36\sqrt{641}$   
 $|QL_2| = \frac{|V \times D|}{|D|} \approx 36.58 \text{ units}$       $|D| = 3\sqrt{69}$

3.  $L: -4x + 3y = 9$       $Q(4, -6)$      See my comments:     She used diff. method which is correct. The method we used is 1)  $-4x + 3y - 9 = 0$

~~$I(0, 3, 0)$       $\vec{IQ} = \langle 4, -9, 0 \rangle$       $N = \langle -4, 3, 0 \rangle$~~   
 ~~$|QL| = \frac{|\vec{IQ} \cdot N|}{|N|} = \frac{|4(-4) - 9(3)|}{5} = \frac{43}{5} \text{ units.}$~~   
2) Substitute 4 for x and -6 for y, we get  $|QL| = \frac{|-16 - 18 - 9|}{\sqrt{16 + 9}} = 43/5$

4. a)  $Q_1(2, 2, 2)$       $Q_2(4, 4, 4)$       $Q_3(8, 7, 6)$   
 $\vec{Q_1Q_2} = \langle 2, 2, 2 \rangle$       $\vec{Q_1Q_3} = \langle 6, 5, 4 \rangle$   
 $N = \vec{Q_1Q_2} \times \vec{Q_1Q_3} = \langle -2, 4, -2 \rangle$   
 equation of plane:  
 $-2(x-2) + 4(y-2) - 2(z-2) = 0$

b)  $P: 2x + y + 2z = 20$       $|N| = 0.025$   
 $N = \langle 2, 1, 2 \rangle$       $0.025 \langle 2, 1, 2 \rangle \Rightarrow N = \langle \frac{2}{120}, \frac{1}{120}, \frac{2}{120} \rangle$   
 $|N| = 3$      3

c)  $P: 2x + y + 2z = 20$       $V = \langle -4, 0, 6 \rangle$   
 $N = \langle 2, 1, 2 \rangle$   
 $N \cdot V = 4 \neq 0 \Rightarrow$  The vectors are not  $\perp \Rightarrow$  You can't draw it inside the plane.

$$d) Q(20, 10, -12.5)$$

$$P: 2x + y + 2z = 20.$$

$$N = \langle 2, 1, 2 \rangle$$

$$I(0, 0, 10)$$

$$\vec{IQ} = \langle 20, 10, -22.5 \rangle$$

$$|QP| = |\vec{IQ} \cdot \vec{N}|$$

$$5 \text{ units}$$

$$|N|$$

$$2\sqrt{2}$$

She used Diff. Method which is correct (but her  $N$  is false, note that

$N = \langle 2, 1, 2 \rangle$ . Our method is

1)  $2x + y + 2z - 20 = 0$  and  $N = \langle 2, 1, 2 \rangle$ . 2) Substitute 20 for  $x$ , 10 for  $y$

and

-12.5 for  $z$ , so  $|QP| = |40 + 10 - 25 - 20| / |N| = 5 / \sqrt{4 + 1 + 4} = 5/3$  units

Hw 10

Q1.  $P_1: 3x + 2y - z = -2$      $P_2: -2x - 2y + 4z = -4$

$N_1 < 3, 2, -1 >$      $N_2 < -2, -2, 4 >$

$N_1 \times N_2$	i	j	k	=	<				
	3	2	-1			2	-1		3
	-2	-2	4			-2	4		-2
									2
									-2

$< 6, -10, -2 >$  intersect.

Let  $z = 0$ .

~~$3x + 2y = -2$~~   
 ~~$-2x - 2y = -4$~~

replace  $\rightarrow 3(-6) + 2y = -2$

$x = -6$

$y = 8$   
 $(-6, 8, 0)$  intersectional point

$(-6, 8, 0) + t < 6, -10, -2 >$

Q2.  $p: -3x + y + z = 21$

$l: x = 2t$

$y = 4t + 10$

$z = 2t + 5$

$D < 2, 4, 2 >$

$-3(2t) + (4t + 10) + (2t + 5) = 21$

$-6t + 4t + 10 + 2t + 5 = 21$

$15 \neq 21$

Can we Draw D inside the plane P?  
 $N = < -3, 1, 1 >$   
 $D \cdot N = 0$ . So yes we can.

This is good question. The line does not lie in the plane, but D can be drawn in the plane!



Q3. L:  $x = 2t + 1$   
 $y = 4t - 2$   
 $z = 5t + 1$

p:  $3x + 2y - z = 4$

Find Q (intersection point).

$$3(2t+1) + 2(4t-2) - (5t+1) = 4$$

$$6t + 3 + 8t - 4 - 5t - 1 = 4$$

$t = 2/3$ . Substitute  $2/3$  for  $t$  in L, we get

~~$(3, -4, -1)$~~   $(7/3, 2/3, 13/3)$

Q4.  $p_1: 3x + 9y - 3z = 12$   $N_1 < 3, 9, -3 >$

$p_2: x + 3y - z = 4$   $N_2 < 1, 3, -1 >$

$$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 3 & 9 & -3 \\ 1 & 3 & -1 \end{vmatrix} = \langle 9(-3) - (3(-3)), 3(-1) - (1(-1)), 3(9) - 13 \rangle$$

$\langle 0, 0, 0 \rangle$

Q5.  $p_1: x + 2y - 5z = 7$

$p_2: -3x - 6y + 15z = -20$

$N_1 < 1, 2, -5 >$

$N_2 < -3, -6, 15 >$

$$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & -5 \\ -3 & -6 & 15 \end{vmatrix}$$

$\cdot 3(1) + 9(6) - 3(0) = 12$

$(1, 1, 0)$

replace in

$x + 3y - z = 4$

$1 + 3(1) - 0 = 4$

$4 = 4 \checkmark$  yes.

$$= \langle \begin{vmatrix} 2 & -5 \\ -6 & 15 \end{vmatrix}, \begin{vmatrix} 1 & -5 \\ -3 & 15 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ -3 & -6 \end{vmatrix} \rangle$$

$\langle 0, 0, 0 \rangle$

$$x + 2y - 5z = 7 \quad (5, 1, 0)$$

$$5 + 2(1) - 5(0) = 7$$

7 = 7 replace in

$$-3x - 6y + 15z = -20$$

$$-3(5) - 6 + 15(0) = -20$$

$$-21 \neq -20 \text{ parallel.}$$

Another way, choose  $y = z = 0$  in  $P_1$ , hence  $x = 7$ . Thus  $(7, 0, 0)$  is in  $P_1$ . Substitute 7 for  $x$ , 0 for  $y$ , and 0 for  $z$  in  $P_2$ , we get  $-21 + 0 + 0 = -20$  (invalid). Thus  $P_1$  parallel to  $P_2$

From: [Leen Agha AlNimer](#)  
To: [Ayman Badawi](#)  
Subject: Solution HW 11  
Date: Wednesday, March 21, 2018 7:21:17 AM

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v.  $f(x) = 12x^4 + 10x^2 - 2018$ , find  $f'(-1)$

$$f'(x) = 48x^3 + 20x - 0$$

$$f'(-1) = 48(-1)^3 + 20(-1)$$

$$f'(-1) = -48 - 20 = -68$$

HW 11

Q1i.  $y = 5x^2 + 12x - 4\sqrt{x} + \frac{4}{x^5}$

$y = 5x^2 + 12x - 4x^{1/2} + 4x^{-5}$

$y' = 10x + 12 - 2x^{-1/2} - 20x^{-6} = 10x + 12 - \frac{2}{\sqrt{x}} - \frac{20}{x^6}$

ii.  $y = x^2 + 10x - 5\sqrt{x}$

$y = x^2 + 10x - 5x^{1/2} + x^{-2/5}$   $y = x^{8/5} + 10x^{3/5} - 5x^{8/10}$

$y' = 2x + 10 - \frac{5}{2}x^{-1/2} + \frac{2}{5}x^{-7/5}$  Now start cooking

$y' = 2x + 10 - \frac{5}{2\sqrt{x}} - \frac{2}{5x^{7/5}}$

iii.  $y = 4x^2(x+3)^2 + 10x - 7$

$y = 4x^2(x^2 + 6x + 9) + 10x - 7$

$y = 4x^4 + 24x^3 + 36x^2 + 10x - 7$

$y' = 16x^3 + 72x^2 + 72x + 10 - 0$

iv.  $f(x) = 8\sqrt{x} + \frac{32}{x}$ , find  $f'(4)$

$f(x) = 8x^{1/2} + 32x^{-1}$

$f'(4) = \frac{4}{\sqrt{4}} - \frac{32}{4^2}$

$f'(x) = 4x^{-1/2} - 32x^{-2}$

$= \frac{4}{2} - \frac{32}{16}$

$f'(x) = \frac{4}{\sqrt{x}} - \frac{32}{x^2}$

$f'(4) = 2 - 2 = 0$

### Question 3

$$\text{let } f(x) = (x^2 + 4x + 3)^{11}$$

i) critical values  $\rightarrow$

$$f'(x) = 0$$

$$f'(x) = 11(x^2 + 4x + 3)^{10} \cdot (2x + 4)$$

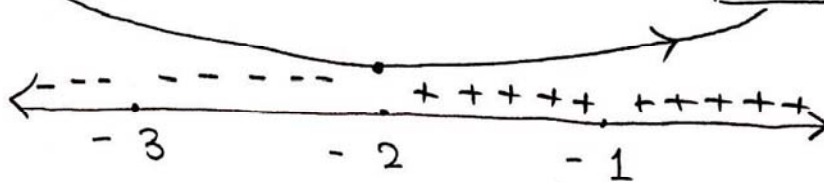
$$(x^2 + 4x + 3) = 0$$

$$\text{or } 2x + 4 = 0$$

$$2x = -4$$

$$\textcircled{1} \frac{(x+1)(x+3)}{x = -1} \quad \textcircled{2} \frac{(x+3)}{x = -3}$$

$$\textcircled{3} x = -2$$



$$f'(-4) < 0$$

$$f'(-2.5) < 0$$

$$f'(-1.5) > 0$$

$$f'(0) > 0$$

ii) \* Increase:  $(-2, \infty)$

\* decrease:  $(-\infty, -2)$

iii) No local max.

Local min at  $x = -2$

iv) sketch

Reem Alali

$$4) f(x) = 3x^2 + \sqrt[3]{2x+1} + 7x, \quad x=0$$

$$\bullet f(0) = 3(0)^2 + \sqrt[3]{2(0)+1} + 7(0)$$

$$f(0) = 1 \quad Q(0, 1)$$

\* eq. of the tangent line:

$$y = mx + b$$

to find  $m$ ,  $\rightarrow f'(0)$

$$f'(x) = 6x + \frac{1}{3}(2x+1)^{-\frac{2}{3}} \cdot (2) + 7$$

$$f'(x) = 6x + \frac{2}{3}(2x+1)^{-\frac{2}{3}} + 7$$

$$f'(0) = 6(0) + \frac{2}{3}(2(0)+1)^{-\frac{2}{3}} + 7$$

$$f'(0) = \frac{23}{3} \rightarrow m = \frac{23}{3}$$

• sub  $x$  &  $y$  ( $Q$ ) to find  $b$  value.

$$y = mx + b \rightarrow 1 = \frac{23}{3}(0) + b$$

$$\boxed{1 = b}$$

\* equation of the tangent line:

$$y = \frac{23}{3}x + 1$$

\* equation of the normal line  $y = mx + b$

$$\frac{23}{3}m = -1$$

$$m = -\frac{3}{23}$$

$$1 = -\frac{3}{23}(0) + b$$

$$\boxed{1 = b}$$

$$\boxed{y = -\frac{3}{23}x + 1}$$

Usem Kashid Alleem  
Homework 12

$$1) y = x(x^2 + 3)^2$$
$$y = x(x^4 + 6x^2 + 9)$$
$$y = x^5 + 6x^3 + 9x$$
$$y' = 5x^4 + 18x^2 + 9$$

$$2) y = 3(2x+1)^{11} + \sqrt[5]{(2x^2+3x-2)} - 7x^4 + 2x^2 - 10$$
$$y = 3(2x+1)^{11} + (2x^2+3x-2)^{1/5} - 7x^4 + 2x^2 - 10$$
$$y' = 33(2x+1)^{10} \cdot (2) + \frac{1}{5}(2x^2+3x-2)^{-4/5} \cdot (4x+3) - 28x^3 + 4x$$

$$3) y = 12(\sqrt{x} + 2x+4)^{10}$$
$$y' = 120(\sqrt{x} + 2x+4)^9 \cdot \left(\frac{1}{2}x^{-1/2} + 2\right)$$

MALAK ALHAWANDEH.

\* If ln is inside  $\rightarrow$  power rule.

\* If outside.  $\rightarrow$  normal.

H.W 15 . LOGARITHMS.

1.  $y = 3 [\ln(2x+7)]^{10}$ .

$3 \times 10 [\ln(2x+7)]^9$ .

(When ln is inside).

$\rightarrow y' = 30 [\ln(2x+7)]^9 \times \frac{2}{2x+7}$ .

2.  $3 \ln(2x+7)^{10}$ .

$30 \times \ln(2x+7) \rightarrow \frac{30(2)}{2x+7} \rightarrow \boxed{\frac{60}{2x+7}}$ .

3.  $y = ((x+1) \ln(3x-1)^3)$

u.v  $\rightarrow u'v + uv'$

$y' = (1) \ln(3x-1)^3 + (x+1) \cdot 3 \ln^2(3x-1) \cdot \frac{3}{3x-1}$ .

$y' = \ln(3x-1)^3 + 3x + 3 \times \frac{3}{3x-1}$ .

4.  $y = x \ln(2x+1)$

when  $x = 2$ .

$y = 2 \ln(2(2)+1)$ .  $y = 3.2$   $(2, 3.2)$ .

$\check{y} = m\check{x} + c$ .

$m = u.v \rightarrow u'v + uv' \rightarrow \ln(2x+1) + x \times \frac{2}{2x+1} \rightarrow \ln(2x+1) + \frac{2x}{2x+1}$

$\ln(2(2)+1) + \frac{2(2)}{2(2)+1} = 2.4 \dots$

$3.2 = 2.4(2) + c$

$3.2 = 4.8 + c$   $c = -1.6$ .

$y = 2.4x - 1.6$ .  
EQUATION OF TANGENT.



## EQUATION OF NORMAL.

$$m = -\frac{1}{2.4} \quad y = mx + c.$$

$$3.2 = \left(-\frac{1}{2.4}\right)(2) + c$$

$$3.2 = -\frac{5}{6} + c \quad c = \frac{121}{30} \quad (\text{or } 4.0333\dots).$$

$$y = -\frac{1}{2.4}x + \frac{121}{30}$$

5.  $y = (x+1) + \ln(3x-1)^3.$

~~$$1 + \frac{9 \ln(3x-1)}{3x-1}$$~~

$$1 + \frac{9 \ln(3x-1)}{3x-1}$$

6.  $y = e^{(2x+1)} \times \ln(7x+2).$   $\rightarrow$  u.v  $\rightarrow$  uv' + uv'

$$y' = e^{(2x+1)} \times 2 \ln(7x+2) + e^{(2x+1)} \times \frac{7}{7x+2}$$

7.  $y = \log(\sqrt{x} + 3x - 1).$   $\rightarrow \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$

↓

$$y = \frac{\ln(\sqrt{x} + 3x - 1)}{\ln(10)} \rightarrow \ln(x^{1/2} + 3x - 1)$$

$$y' = \frac{\frac{1}{2}x^{-1/2} + 3}{x^{1/2} + 3x - 1} \times \frac{1}{\ln(10)}$$

HW : ①  $\int \frac{2+4x}{x^{12}} dx = \int x^{-12} (2x+4x) dx$  Rana Hegab  
Muhammad Farouqi  
Olga Davidovskaya

$$= \int 2x^{-12} + 4x^{-11} dx = \frac{-2}{11} x^{-11} + \frac{-4}{10} x^{-10} + C$$

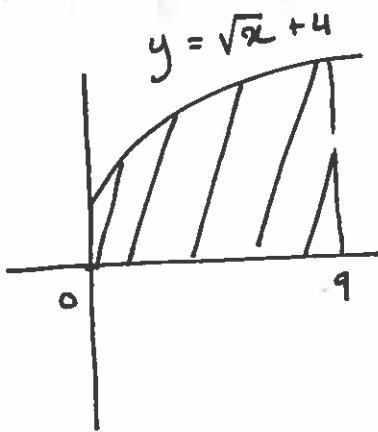
$$= \frac{-2}{11x^{11}} - \frac{4}{10x^{10}} + C$$

②  $\int \frac{2x^2+2}{x^3+3x+7} dx = \frac{2}{3} \int \frac{3}{3} \cdot \frac{x^2+1}{x^3+3x+7} dx$

$$= \frac{2}{3} \int 3(x^2+1)(x^3+3x+7)^{-1} dx = \frac{2}{3} \ln(x^3+3x+7) + C$$

③  $\int (4x^3 + e^x) e^{(x^4 + e^x)} dx = e^{(x^4 + e^x)} + C$

④



$$\int_a^b f(x) dx = \int_0^9 \sqrt{x} + 4 dx$$

$$= \int_0^9 x^{\frac{1}{2}} + 4 dx = \left. \frac{2}{3} x^{\frac{3}{2}} + 4x \right|_0^9$$

$$= \left( \frac{2}{3} (9)^{\frac{3}{2}} + 4(9) \right) - \left( \frac{2}{3} (0)^{\frac{3}{2}} + 4(0) \right)$$

$$= \left( \frac{2}{3} (9)^{\frac{3}{2}} + 36 \right) = 18 + 36 = 54 \text{ unit}^2$$

$$\textcircled{5} \int w^2 (2w+1)^2 dw = \int w^2 (4w^2 + 4w + 1) dw$$

$$= \int (4w^4 + 4w^3 + w^2) dw = \frac{4}{5} w^5 + \frac{4}{4} w^4 + \frac{1}{3} w^3 + C$$

$$= \frac{4}{5} w^5 + w^4 + \frac{1}{3} w^3 + C$$

$$\textcircled{6} \int \frac{(4x+2)}{\sqrt{x^2+x+1}} dx = \int (4x+2)(x^2+x+1)^{-\frac{1}{2}} dx$$

$$= 2 \int (2x+1)(x^2+x+1)^{-\frac{1}{2}} dx = \frac{2}{\frac{1}{2}} (x^2+x+1)^{\frac{1}{2}} + C$$

$$= 4(x^2+x+1)^{\frac{1}{2}} + C$$